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[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 4470

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Unique Paper Code : 32357502

Name of the Paper : DSE-1 Mathematical  
Modelling and Graph Theory

Name of the Course : B.Sc. (H) Mathematics –  
(LOCF)

Semester : V

Duration : 3 Hours Maximum Marks : 75

**Instructions for Candidates**

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All questions are compulsory.
3. Attempt any three parts from each question.

1. (a) (i) Determine whether  $x = 0$  is an ordinary point, a regular singular point or an irregular singular point of the differential equation

$$x^2y'' + (6 \sin x)y' + 6y = 0.$$

- (ii) Find the Laplace transform of the function  
 $f(t) = \sin 3t \cos 3t$

P.T.O.

(iii) Find the inverse Laplace transform of the function  $F(s) = \frac{9+s}{4-s^2}$ . (6)

(b) Use Laplace transforms to solve the initial value problem :

$$x'' + 6x' + 25x = 0; x(0) = 2, x'(0) = 3 \quad (6)$$

(c) Find two linearly independent Frobertius series solutions of (6)

$$4xy'' + 2y' + y = 0$$

(d) Find general solutions in powers of  $x$  of the differential equation. State the recurrence relation and the guaranteed radius of convergence.

$$5y'' - 2xy' + 10y = 0 \quad (6)$$

2. (a) Explain Linear Congruence method for generating random numbers. Does this method have any drawback? Explain with the help of an example. (6)

(b) Use of Monte Carlo simulation to approximate the area under the curve  $y = \cos x$  over the interval  $-\pi/2 \leq x \leq \pi/2$ , where  $0 \leq \cos x \leq 2$ . (6)

(c) Using algebraic analysis, solve the following:

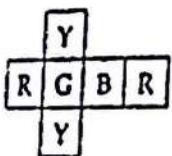
$$\begin{aligned} \text{Maximize :} & \quad x + 2y \\ \text{subject to} & \quad 5x + 2y \leq 10 \\ & \quad 2x + 3y \leq 6 \\ & \quad x_1, x_2 \geq 0. \end{aligned} \quad (6)$$

- (d) Consider a small harbor with unloading facilities for ships, where only one ship can be unloaded at any time. The unloading time required for a ship depends on the type and the amount of cargo. Below is given a situation with 5 ships:

	Ship 1	Ship 2	Ship 3	Ship 4	Ship 5
Time between successive ships	20	30	15	120	25
Unload time	55	45	60	75	80

Draw the timeline diagram depicting clearly the situation for each ship. Also determine length of longest queue and total time in which docking facilities are idle. (6)

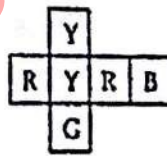
3. (a) Find the solution to the four-cubes problem for the following set of cubes. (6)



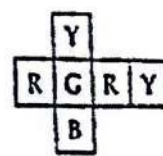
cube 1



cube 2



cube 3



cube 4

- (b) Define semi-Eulerian trail. Prove that a bipartite graph with an odd number of vertices is not Hamiltonian. (6)
- (c) Prove that there is no Knight's tour on a  $7 \times 7$  Chessboard. (6)

P.T.O.

(d) State Handshaking lemma. Use it to prove that in any graph, the number of vertices of odd degree is even. (6)

4. (a) Use the factorization :

$$s^4 + 4a^4 = (s^2 - 2as + 2a^2) (s^2 + 2as + 2a^2)$$

and apply inverse Laplace transform to show that :

$$L^{-1} \left\{ \frac{s}{s^4 + 4a^4} \right\} = \frac{1}{2a^2} \sinh at \sin at \quad (7)$$

(b) Using Simplex method, solve the following linear programming problem :

Maximize  $z = 5x_1 + 3x_2$

subject to  $-3x_1 - 5x_2 \geq -15$

$$5x_1 + 2x_2 \leq 10$$

$$x_1, x_2 \geq 0. \quad (7)$$

(c) (i) State Ore's Theorem. (2)

(ii) Show that  $L \{t \cos(kt)\} = \frac{s^2 - k^2}{(s^2 + k^2)^2} . \quad (5)$

(d) Define Cube graphs. Write the number of vertices and number of edges in a cube graph  $Q_k$ . Draw  $Q_1, Q_2$  and  $Q_3$ . (7)

(1000)